Department of Mathematics, University of Toronto
MAT224H1S - Linear Algebra II
Winter 2016

Lectures & Administrative Information

Section	Time	Lecture Room	Instructor	Office
L0101	T1-3, W1	SS 2118, SS 2117	F. Herzig	BA 6186
L0201	W3-5, R3	MP 102	S. Uppal	PG 112
L0301	R11-1, F11	MP 102	I. Biborski	FI 340
L0401	T3, R6-8	MP 203, MP 102	K. Leung	BA 6256
L5101	T6-9	SF 1105	A. Garcia-Raboso	BA 6172

Course Coordinator: S. Uppal.

Email: uppal@math.utoronto.ca.

Office hours: Tuesdays 3:10-4:00pm, Wednesdays 5:10-6:00pm, Thursdays 4:10-6:00pm or by appointment. If you would like to book an appointment outside my regularly scheduled office hours, please send me an email indicating the times you are available to meet. Also, please give at least 24 hours notice for appointments so that there is suitable time to make arrangements.

Office hours for all instructors will be posted on Blackboard by the end of the first week of classes.

Email Policy & Etiquette

We will respond to emails as soon as possible, usually within 24-48 hours (except on weekends). Several days before an exam is always a particularly busy time and it may take us longer to respond. If your situation is urgent, it's best to speak with us in person either before or after class or during office hours.

- **Put MAT224 in the subject line, use your UofT email, and always identify yourself**.
- **Be specific**. We're better able to help you the more specific you are. If your question is complex or lengthy and requires multiple back-and-forth emails, we will ask you to to come office hours, or make an appointment, instead.
- **Check the syllabus and Blackboard first**. If the answer to your question(s) is available in the syllabus or on Blackboard, we will not respond to your email.
- **Be professional**. Please use an appropriate tone, level of formality, and review what you've written before sending your email. Email, in the context of the class and communication with instructors, is professional correspondence and we expect you to treat it as such.

Brief Course Description & Goals

This second course in Linear Algebra expands the breadth and depth of the material from MAT223 - Linear Algebra I, which is a prerequisite for the course. We will analyze particular sets (vector spaces) and special mappings/functions between these sets (linear transformations), and classify all such maps (diagonal and canonical forms). The motivation for much of what we'll do comes from what we covered in MAT223. Always keep in mind that in linear algebra **concepts are as important as computations**.

We recommend you review the material from MAT223 - solving systems of linear equations, subspaces of $\mathbb{R}^n$, span, linear independence and dependence, basis, dimension, rank, column space, null space, projections, and diagonalization - particularly if it's been more than one semester since you took it. We will talk about some of these topics in a more general setting.

It is hoped that by the end of the course you will have

- become fluent in linear algebra and some of its applications.
- become comfortable reading and understanding precise mathematical statements, definitions, and proofs.
- sharpened your problem solving, reasoning, and writing skills.

You will see some superb material in this course. If you run into some trouble along the way, please do not hesitate to contact your instructor or TA for help. See the weekly class schedule below for a full list of topics covered.

Textbook and Reading Material

Required: David B. Damiano & John B. Little: *A Course in Linear Algebra.* ISBN: 978-0-486-46908-9. Dover publications.

This textbook is the best textbook for the course given the content we need to cover and various academic backgrounds of the students enrolled. The textbook has an easy-going, conversational style but doesn't lack rigour. There may be a couple of times we cover material in a more general setting than the textbook (such as Chapter 4) , and others where we don't go in as much depth (such as Chapter 6). Not attending lectures and attempting to learn strictly from the textbook may be problematic. The one aspect where the textbook could be improved is in the number of exercises in each section; there could be more. To compensate, we will give you additional set of problems every week to work on and you will also see examples in class. If you find you need more practice, please ask and we can suggest alternative texts/resources other than those below. There is no solutions manual for this book but there are solutions in the back of the textbook for some exercises.

Recommended: Sheldon Axler: *Linear Algebra Done Right, 3rd ed.* ISBN: 978-3-319-11079-0. Springer.

It begins slightly in more general setting than Damiano & Little but this could, and may be in the future, the textbook for this course.

Recommended (advanced): Stephen H. Friedberg, Arnold J. Insel, Lawrence E. Spence: *Linear Algebra, 4th ed.* ISBN:978-0-130-08451-4. Pearson Education

This is the textbook used for the specialist linear algebra MAT240/MAT247. A solid, well written text, with good problems but a little advanced for this course. For those of you looking to delve a little deeper, this is the text to read.

Course Webpage

The website for this course is accessible through http://www.portal.utoronto.ca

Please check the website frequently for course announcements and materials. All announcements posted are considered to have been announced to the class and not having read or seen an announcement **is not** an accepted reason for not following guidelines or missing deadlines. You may configure your preferences on portal to receive email notification as soon as an announcement has been posted.

Marking Scheme

Your final grade will be calculated by the following formula:

- Writing Assignments - 10% of your final grade.
- Midterm Exam I & II - 50% of you final grade (combined). The higher of Midterm I/II will count for 30% of your final grade; the lower 20%.
- Final Exam - 40% of your final grade.

Your raw scores for each piece of term work will be recorded on Blackboard. Please check regularly that your marks have been recorded accurately. If there are any discrepancies, please email the course coordinator within two weeks of the grade having been posted at uppal@math.utoronto.ca. You will, of course, need evidence that your grade is not recorded correctly.

Course Components & Assignments

Lectures

You will get the most out of lectures if you come to really engage with the material as opposed to just taking notes (or not). Try to make sense of individual topics and their connections to other topics and how to translate seemingly abstract concepts into simple terms. If you do choose to take notes, I suggested re-writing and revising your notes the same day, while concepts are still fresh in your mind.

While cell phones are not prohibited in lecture, **recording or taking pictures in lecture is strictly prohibited without the consent of your instructor**. Please ask before doing.

Tutorials

Every student should be registered in one tutorial section. You may register in one of the tutorial time slots through ROSI/ACORN by the end of the first week of classes. After that, look for and follow the instructions on Blackboard about enrolling in a tutorial. By the end of the second week of classes tutorial groups and locations will be posted on the course website.

Tutorials begin the 3rd week of classes. Tutorials are an integral part of the course and should be regarded as just as important as lectures. During your tutorials your TA will discuss 'Tutorial Problems' which will

be posted on the course website each Friday and will be discussed in your tutorial the following week. The problems are meant to develop your skills, deepen your understanding, and to help prepare you for the exams. **Up to half of the material for the midterm exams will come from the Tutorial Problems**, possibly with some numbers changed. As a result, **solutions to the Tutorial Problems will not be posted**. It's important then that you practice early and often, identify what you're having the most trouble with and ask questions.

As with lectures, **recording or taking pictures in tutorials is strictly prohibited without the consent of your TA**. Please ask before doing.

Midterms & Final Exam

There will be two 1hr 50min minute midterm exams and one 3hr final exam common to all sections of the course. Each midterm will emphasize material not already tested but may build on previous material. The final exam will be cumulative.

The dates of the midterms are:

- Midterm I - Friday February 12, 12:10-2:00pm. An early sitting is available from 10:10am-12:00pm for those with a legitimate conflict.
- Midterm II - Friday March 18, 12:10-2:00pm. An early sitting is available from 10:10am-12:00pm for those with a legitimate conflict.

The date of the final exam is to be determined by the Faculty of Arts & Science but will be scheduled sometime April 12-29.

If you have a legitimate conflict and need to register for the early sitting of the exams, please read and follow the instructions that will be posted on Blackboard roughly two weeks before the date of each midterm.

Each midterm exam and the final exam may contain multiple choice questions, short answer questions, theory questions, precise definitions and statements of theorems. Exact details about exam content and format will be posted on the website roughly two weeks before the date of each midterm. Midterms and the final exam are closed book and no calculators or other aids are allowed.

There will be no make-up exams.

Writing Assignments

The purpose of the assignments is to improve your writing skills - writing is not the same as writing well - and understanding what constitutes good mathematical exposition. Writing well is key to your future success, both academically and in industry. You'll need to sell yourself when applying for jobs (resume, cover letter) and once successful, be able to explain decisions you make. Finding a solution is not the goal - it's explaining how **you** got **your** solution and why your solution correct.

Each assignment will consist one one or two problems, possibly with multiple parts. The problems may include topics we've covered in lecture; may include topics we've not covered but you have the skills to solve; or may be a combination of the two.

Each assignment will be assessed with detailed feedback intended to help you understand the gap between your solution (present position) and a perfect solution (desired goal). You will receive either a score of 1 for a neatly written, clear attempt to solve the problem(s); or 0 for no clear attempt to solve the problem(s) or a messy, scribbled solution. Presentation counts, as does content, though you don't need to have a perfect solution to get a perfect score. The idea is to give you some freedom to play and explore; to be creative and to make mistakes. The more you put into the assignments, the more you'll get out of them.

There will be more or less weekly writing assignments throughout the term. Each assignment will be posted on Friday and due the following Friday - i.e. you have one week to complete each assignment. The first assignment will be posted Friday January 22. There will be no assignments due the week of your midterm exams. There will be 8 assignments in total, but only your best 7 will count. Each assignment is worth an equal value toward your writing assignment grade.

Warning: The assignments are generally on the short side and low stakes. If you do nothing else every week but the assignments, you'll struggle with this course. You need to practice writing solutions to more than one or two problems per week to be successful.

Tips to do well

- Attend every lecture and tutorial.
- Come to lecture and tutorial prepared. For lectures, this means reviewing the material from the previous week, and reading the relevant sections in the textbook beforehand. Be active while reading - write definitions and statements of theorems and note any concepts that are unclear and any questions you may have. You can then either bring them up in lecture or see if the lecture has answered your questions. For tutorials, this means attempting the Tutorial Problems in advance. The key is to discover what you do and don't know and where there are gaps in your understanding. Once you look up a solution, or have someone show you a solution, you lose out on this valuable insight.
- Practice, practice, practice. Learning linear algebra is like learning a new language, to master it requires consistent practice. Once you've read the textbook and reviewed your notes, you won't gain much by re-reading them ad nauseum. Practice problems as much as you can. Practice early and often rather than cramming in short bursts.
- Learn, don't memorize. Learning is an active process; memorizing is passive.
- Form study groups. You will learn from one another, through both your expertise and your mistakes.
- Ask questions. Lots of them. If you're stuck on a problem and don't know where to begin, a good starting point is to identify the keywords and ask yourself "what does this mean?".
- Complete **all** the term work. Consistently, the top marks for the course are earned by students who don't defer any exams and complete all the assignments, even though we drop your lowest assignment score.
- Average 8 hours (480 minutes) of study a week for this course - 1/5th of a full time job. Being engaged in lectures and tutorials is 200 minutes and gets you almost halfway there. The remaining time should be spent mainly practicing problems.

Course Policies

Missed Exams

- You will be assigned a grade of 0 for any midterm exam you do not write unless you submit a University of Toronto Verification of Student Illness or Injury form - http://www.illnessverification.utoronto.ca/getattachment/index/Verification-of-Illness-or-Injury-form-Jan-22-2013.pdf.aspx - within one week after the date of the exam.
- The form must have all the required fields properly filled out and it must list the doctor's OHIP number.
- The form must clearly state that on the date of the exam you were unable to write. Accordingly, it's expected that you will have met your doctor on the date of the exam. Illness before the exam is not sufficient grounds for not writing the exam nor is the claim that you would have performed "sub-optimally". The form **cannot just report that you told the doctor after-the-fact that you were ill previously**.
- The form must be original and completed by a qualified medical doctor - **not** an acupuncturist, chiropractor, or other health care professional.
- Once you submit your form, it will be reviewed before it will be accepted. Part of the review process may include following up with your doctor, your college registrar, or the undergraduate chair of the math department. It is an academic offence to feign illness to miss an exam.
- If you do miss either Midterm I or II for a legitimate reason that you can document, and your documentation is accepted, then your final exam will account for 65% of your final grade, and the midterm exam you do write will account for 25% of your final grade.

Writing Assignments

- Late assignments will not be accepted nor will any extensions be given for any reason including, but not limited to, not having an internet connection, technical issues (problems with your browser, power outages, problems with your laptop, software issues, etc). Start and finish assignments early and don't wait until the last minute to submit them - you do so at your own risk.
- Medical notes will not be accepted for missing an assignment.
- Under no circumstances will the weight of any assignment(s) be transferred to the final exam.

Academic Resources

Accessibility Needs

The University of Toronto is committed to accessibility. If you require accommodations for a disability, or have any accessibility concerns about the course, the classroom or course materials, please contact Accessibility Services - http://www.studentlife.utoronto.ca/ - as soon as possible.

Writing and English Language Instruction

For information on campus writing centres and writing courses, please visit http://www.writing.utoronto.ca/.

FREE English language instruction with the ELL Program will start in Winter 2016. The Communication Cafe offers drop-in discussions, presentations, and debates, along with learning about Canadian culture starting January 11, 2016 - no registration necessary. Sessions are facilitated by writing centre instructors. Also, registration is open NOW for Reading eWriting, online writing practice with a writing centre instructor starting January 15, 2016. For information about the English Learning Language (ELL) program and its services, please visit

http://www.artsci.utoronto.ca/current/advising/ell.

Other Resources

Student Life Programs and Services: http://www.studentlife.utoronto.ca

Academic Success Centre: http://www.studentlife.utoronto.ca/asc

Health and Wellness Centre: http://www.studentlife.utoronto.ca/hwc

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Academic Integrity

Academic integrity is fundamental to learning and scholarship at the University of Toronto. Participating honestly, respectfully, responsibly, and fairly in this academic community ensures that the U of T degree that you earn will be valued as a true indication of your individual academic achievement, and will continue to receive the respect and recognition it deserves.

Familiarize yourself with the University of Toronto's Code of Behaviour on Academic Matters

http://www.governingcouncil.utoronto.ca/policies/behaveac.htm.

It is the rule book for academic behaviour at the U of T, and you are expected to know the rules.

The University of Toronto treats cases of academic misconduct very seriously. All suspected cases of academic dishonesty will be investigated following the procedures outlined in the Code. The consequences for academic misconduct can be severe, including a failure in the course and a notation on your transcript. If you have any questions about what is or is not permitted in this course, please do not hesitate to contact me. If you have questions about appropriate research and citation methods, seek out additional information from me, or from other available campus resources like the U of T Writing Website. If you are experiencing personal challenges that are having an impact on your academic work, please speak to me or seek the advice of your college registrar.

Weekly Class Schedule

Your instructor may be slightly ahead or behind this schedule. This schedule is subject to change.

Week 1 beginning January 11.
Lecture: Vector Spaces, Subspaces.
Section 1.1
Section 1.2

Week 2 beginning January 18.
Lecture: Linear Combinations, Linear Dependence and Linear Independence.
Section 1.3
Section 1.4

Note: Section 1.5, Solving Systems of Linear Equations, is material from MAT223 that you should review on your own.

Week 3 beginning January 25. **Tutorials begin**.
Lecture: Bases and Dimension. Linear Transformations.
Section 1.6
Section 2.1

Week 4 beginning February 1.
Lecture: Linear Transformations between Finite Dimensional Vector Spaces, Kernel & Image.
Section 2.2
Section 2.3

Week 5 beginning February 8. **Midterm Exam I**
Lecture: Dimension Theorem, Applications of Dimension Theorem, Composition of Linear Transformations.
Section 2.4
Section 2.5

Week 6 beginning February 22.
Lecture: Inverse of Linear Transformation, Isomorphism, Change of Basis.
Section 2.6
Section 2.7

Week 7 beginning February 29.
Lecture: Eigenvalues & Eigenvectors, Diagonalizability.
Section 4.1
Section 4.2

Week 8 beginning March 7.
Lecture: Fields, Vector Space over a Field, Geometry in a Complex Vector Space.
Section 5.1
Section 5.2

Week 9 beginning March 14. **Midterm Exam II**
Lecture: Triangular Form, Canonical Form for Nilpotent Mappings.
Section 6.1
Section 6.2

Week 10 beginning March 21.
Lecture: Jordan Canonical Form, Computing Jordan Form.
Section 6.3
Section 6.4

Week 11 beginning March 28.
Lecture: Geometry in $\mathbb{C}^n$, Orthogonal Projections & Gram-Schmidt.
Section 4.3
Section 4.4
Section 5.3

Week 12 beginning April 4.
Lecture: Symmetric Matrices, Spectral Theorem.
Section 4.5
Section 4.6
